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2 **Title:** Estimating small proportions
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8 **Introduction** 9

10 High statistical power is necessary when attempting to estimate the contribution of stocks which
11 contribute at small proportion to the mixture (e.g. <0.05) in order to detect the presence of these
12 stocks. Along with detecting presence/absence, obtaining unbiased estimates is also important.
13 In other words, we are looking for methods to increase the accuracy and precision of estimates of
14 stocks in mixtures that appear in low proportions. Generally, statistical power is generated
15 through increasing sample sizes within strata; however this is often not an option.
16

17 One way to increase power, when faced with several samples of fixed sample size, is make use
18 of a stratified design. However, stratifying means that we must increase the scope of our
19 estimate. For example, consider the contribution made by North Peninsula stocks of sockeye
20 salmon to the harvest in the Ugashik District over a three-year period. The current sampling plan
21 for this district identifies four temporal strata per year. We could provide a separate estimate for
22 each temporal stratum, a separate estimate for each year, or a single estimate over all years and
23 strata. As we broaden the scope of the estimate, we improve precision and accuracy. Our
24 purpose here is to demonstrate this improvement with a simulated example. The North
25 Peninsula/Ugashik scenario was chosen for this example because there is much genetic overlap
26 between stocks of sockeye salmon spawning within the North Peninsula and Ugashik districts.
27

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Methods

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31 In the Ugashik District in 2008, the estimated composition of the commercial catch of sockeye
32 salmon in all four strata was consistently 85-90% Ugashik fish, 10-15% Egegik fish, and minor
33 contributions from other stocks (Tim Baker, personnel communication). The total harvest in
34 2008 ranged from 69,000 to 446,000 fish with an average of 250,000 and a standard deviation of
35 154,000. We assumed 2008 was a typical fishing season in the Ugashik District and composition
36 and harvest numbers from this year were used as a model for this simulation.

37

38 For each of three years, mixtures for four temporal strata were generated in proportions similar to
39 those estimated in the Ugashik District in 2008, with the contribution from North Peninsula set at
40 1.1% for all samples (Table 1). Each mixture was given a sample size of N=380. To generate
41 each mixture, fish were removed from baseline populations and the remaining baseline was used
42 to resolve the mixture. A total of 3 (years) X 4 (strata/year) = 12 (strata) mixtures were
43 generated. Harvest for each stratum in each year was drawn from a normal distribution using
44 the observed mean and standard deviation from 2008 (Table 2).

45

46 All mixtures were analyzed with an implementation of the Bayesian mixture model (Pella and
47 Masuda 2001) in WinBUGS (Spiegelhalter et al. 2006) using a flat prior. One chain was run for
48 25,000 iterations, burning the initial 5,000. The resulting posterior outputs were read into R
49 using the CODA feature (Plummer et al. 2006). All estimates were rounded to the nearest 1/10
50 of 1%.

51

52 To estimate the contribution of North Peninsula fish, three levels of summaries (posterior means
53 and 90% Bayesian confidence intervals, hereafter referred to as confidence intervals) were
54 calculated: 1) a separate estimate for each stratum in each year; 2) a broader estimate combining
55 all strata within each year; and 3) a single grand estimate combining all years and strata.

56

57 Summaries for each stratum in each year were calculated by simply taking the mean and
58 quantiles of the posterior outputs. Strata were combined into yearly estimates by weighting them
59 by their respective harvests according to the following equation:

60

61

$$p_y = \frac{\sum_{i=1}^4 H_{y,i} p_{y,i}}{\sum_{i=1}^4 H_{y,i}} .$$

62

63 Where $H_{y,i}$ is the harvest in year y and stratum i ; $p_{y,i}$ is the proportion of North Peninsula fish in
64 year y and stratum i ; and p_y is the overall proportion of North Peninsula fish in year y . To
65 calculate confidence intervals for p_y , its distribution was estimated via Monte Carlo by re-
66 sampling the posterior output from each of the constituent strata and applying the harvest to the
67 draws according to the above equation.

68

69 Similarly, all years were combined by weighting the yearly proportions by the yearly total
70 harvests.

71

72

73 **Results**

74

75 The posterior means and confidence intervals for all three levels are shown in Table 3. For the
76 individual strata (level 1), the estimates tend to be noisy with wide confidence intervals, all of
77 which contain zero when rounded to the nearest one-tenth of one percent. Histograms of the
78 posterior outputs from the first year reveal distributions with large modes at or near zero and
79 long, diffuse tails extending well beyond the mean (Figure 1).

80

81 The yearly estimates were better behaved with tighter confidence intervals, one of which
82 excludes zero (Table 3). The posterior distribution of the first yearly proportion is bi-modal,
83 with one mode near zero and the other mode near the true value of 1.1% (Figure 2).

84

85 The estimated grand proportion over all years is very near the true value 1.1% and the tight
86 confidence interval excludes zero (Table 3). The posterior distribution is a very well shaped uni-
87 modal distribution whose mode is near 1.1% (Figure 3).

88

89

90 **Discussion**

91

92 Preliminarily, these results appear to give promise to the task of accurately and precisely
93 estimating small proportions, as long a single overall estimate is acceptable. An obvious caveat
94 of this exercise is that there were always 4 North Peninsula fish in every 380-fish mixture,
95 whereas in reality, this proportion would vary across samples if the fishery actually caught 1.1%
96 North Peninsula fish. Also, we failed to fully examine the benchmark scenario of 0.0% North
97 Peninsula fish to see if an overall estimate would exclude zero. Initial explorations show a small,
98 but positive estimate when the true contribution is 0.0%, as is typical of MSA.

99

100 Another approach under consideration is to simply pool all the samples; not for the purpose of
101 estimating stock proportions, but rather, for the detection of North Peninsula fish. Detection can
102 be ascertained via confidence intervals, or possibly model selection techniques involving either
103 Bayes factors or deviance information criteria (DIC) that has been adapted specifically towards
104 mixture models. Establishing presence/absence of North Peninsula fish can aid in the assessment
105 of the validity of estimates for small contributions.

106

107 A further approach is to analyze several related mixtures simultaneously in a hierarchical setting.
108 In this framework, the prior parameters for the stock proportions would themselves be given a
109 prior distribution that relates the stock proportions from one mixture to the stock proportions of
110 other mixtures and to covariates. Some potential covariates include proximity of the stocks to
111 the fishery, time of the year, magnitude of escapement, results from the Port Moller test fishery,
112 scale patterns or age distributions, etc. These models can improve estimation for any one
113 mixture by borrowing strength from the other mixtures and the covariates. Explorations of these
114 techniques in the current context, as well as others, have been very promising.

115

116

117 **Future Analyses**

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- 119 1. Continue the analysis with true contributions of North Peninsula fish that equal {0.00,
120 0.02, and 0.05}.
- 121 2. Repeat the entire analysis with example stocks that are genetically distinct.
- 122 3. Investigate Bayesian model selection techniques with respect to the presence of small
123 contributions in large samples through the use of confidence intervals, Bayes factors, and
124 DIC.
- 125 4. Develop hierarchical models, with covariates, using known mixtures in realistic
126 proportions. Preceding this exploration would be the identification of covariates that
127 improve explanation of stock proportions.
- 128 5. Replicate all analyses multiple times.

129

130

131 **Literature Cited**

132

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145 **Document 3: Estimating small proportions.**

146 This is a good study of the tradeoffs between detail and uncertainty: the smaller the
147 spatial/temporal scale examined, the less certain the estimate of the interception rate of the stock.
148 It would be useful to clarify two important points. First, there are two general sources of
149 uncertainty in these analyses: A) uncertainty in identifying stock of origin of fish in the sample
150 from the fishery; B) uncertainty in extrapolating from the sample to the entire fishery. The
151 second point is that uncertainty A is the only portion that improved genetic methods can address;
152 uncertainty B is not due to a limitation of GSI but rather to inescapable statistical realities.

153 The authors give a good discussion of the limitations of their work. The fixed number of
154 N. Peninsula fish in the trials means the uncertainty was underestimated, but the pattern of more
155 accuracy when strata are collapsed still holds. Another item for consideration is the possibility of
156 overdispersion in the data due to a variety of biological processes and difficulties in obtaining a
157 completely random sample.

158 A hierarchical framework for analyses is suggested. This could be a great idea – samples
159 from a stratum in one year could have information that could improve estimates from the same
160 stratum in other years. However, the variable assumed to have a hierarchical structure needs
161 careful consideration. On biological grounds, it's reasonable to expect similar fractions of a
162 specific population will be in a fishing district each year. However, the fraction this represents of
163 the fish in the district will vary proportionally to the abundance of the source stock and inversely
164 with the abundance of the other stocks that also frequent the district. It may not be optimal to
165 assume, for example, that the proportion of the catch in the Ugashik district of N. Peninsula
166 origin fits a hierarchical model.

167 We'd like to see these analyses focused more closely on questions of concern to
168 managers and resource users. The current focus of the simulations, on the ability to detect and
169 estimate the contribution of stocks that constitute a small fraction of the catch, is useful but could
170 be made more so. For most management concerns, I think the number of fish intercepted will be
171 more relevant than the fraction of the catch they constitute.

172 For instance, those whose stocks are potentially intercepted are interested in whether the
173 fishery is intercepting a 'large' portion of their stock. 'Large' needs to be defined in terms of its
174 effect on the intercepted stock. Relevant simulations should focus on whether a 'large'
175 interception can be detected and its magnitude reliably estimated. These users are also interested
176 in reducing this interception. Thus, identifying the spatial and temporal distribution of this
177 interception is also important.

178 Conversely, the concern of those participating in the interception fishery is having their
179 fishery unnecessarily restricted. Simulations focused on the probability of estimating a 'large'
180 interception when in fact the interception is 'small' would be most relevant.

181

182 [Unedited comments from "Panel comments October 2009.doc" related to Technical Document 3.]

183 Table 1. Compositions of generated mixtures by stratum in each of three years. Compositions
 184 resemble those estimated in the 2008 Ugashik District fishery.
 185

Region	Percentage			
	Stratum 1	Stratum 2	Stratum 3	Stratum 4
North Peninsula	1.1	1.1	1.1	1.1
Ugashik	90.0	86.8	86.8	84.2
Egegik	8.9	12.1	12.1	14.7
Naknek	0.0	0.0	0.0	0.0
Alagnak	0.0	0.0	0.0	0.0
Kvichak	0.0	0.0	0.0	0.0
Nushagak	0.0	0.0	0.0	0.0
Wood	0.0	0.0	0.0	0.0
Igushik	0.0	0.0	0.0	0.0
Togiak	0.0	0.0	0.0	0.0
Other	0.0	0.0	0.0	0.0

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WASSIP Technical Document 2: Temporal variation in baselines

188 Table 2. Simulated harvest (X 10,000) by year and stratum. Harvests were drawn from a normal
189 distribution using the mean and standard deviation observed in the 2008 Ugashik District fishery.
190

Stratification		Harvest
Year 1	Stratum 1	7.5
	Stratum 2	33.8
	Stratum 3	28.1
	Stratum 4	19.9
	Yearly	89.3
Year 2	Stratum 1	25.9
	Stratum 2	24.6
	Stratum 3	37.4
	Stratum 4	43.5
	Yearly	131.4
Year 3	Stratum 1	14.9
	Stratum 2	39.8
	Stratum 3	43.0
	Stratum 4	16.2
	Yearly	113.9
	Overall	334.6

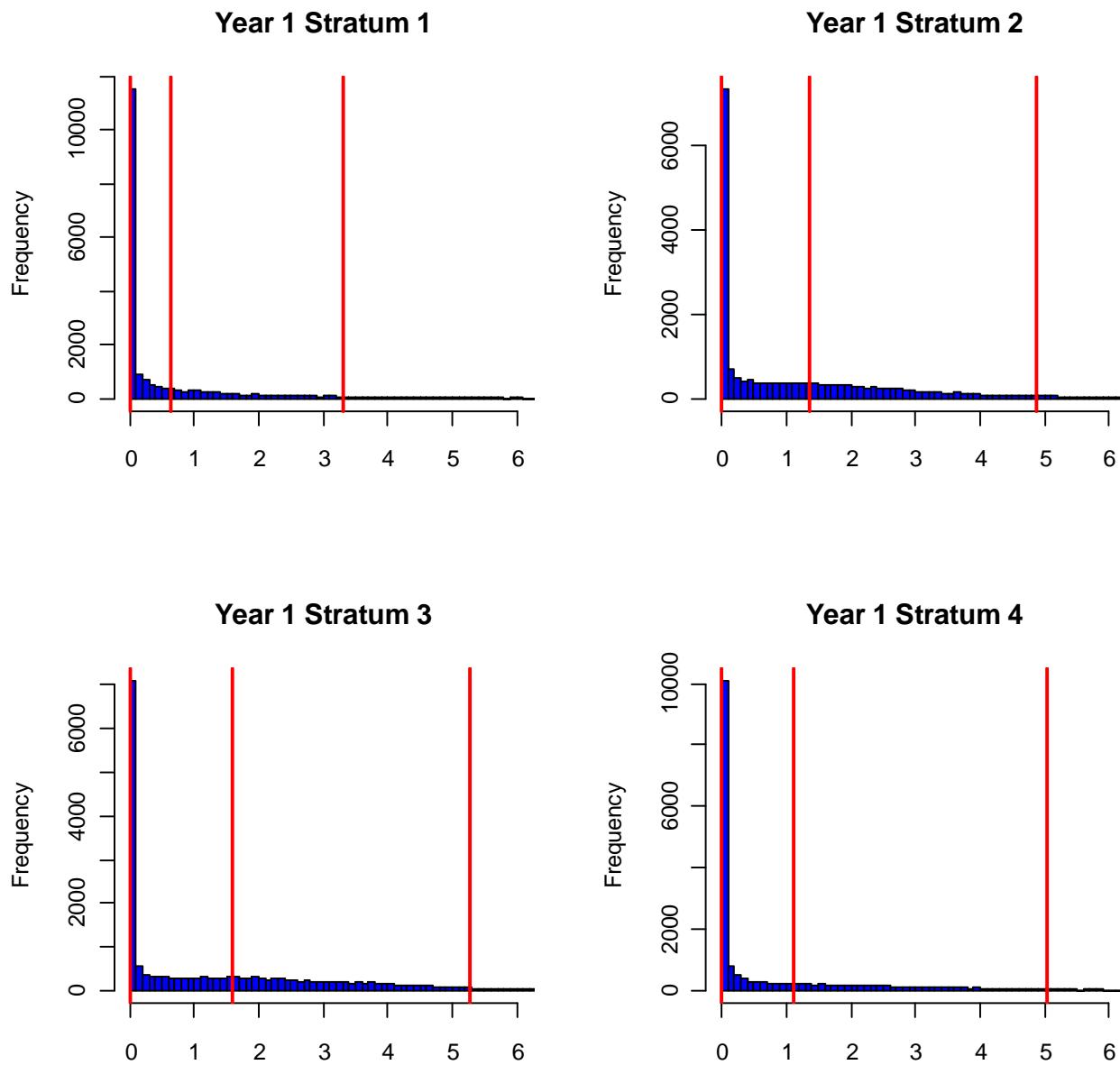
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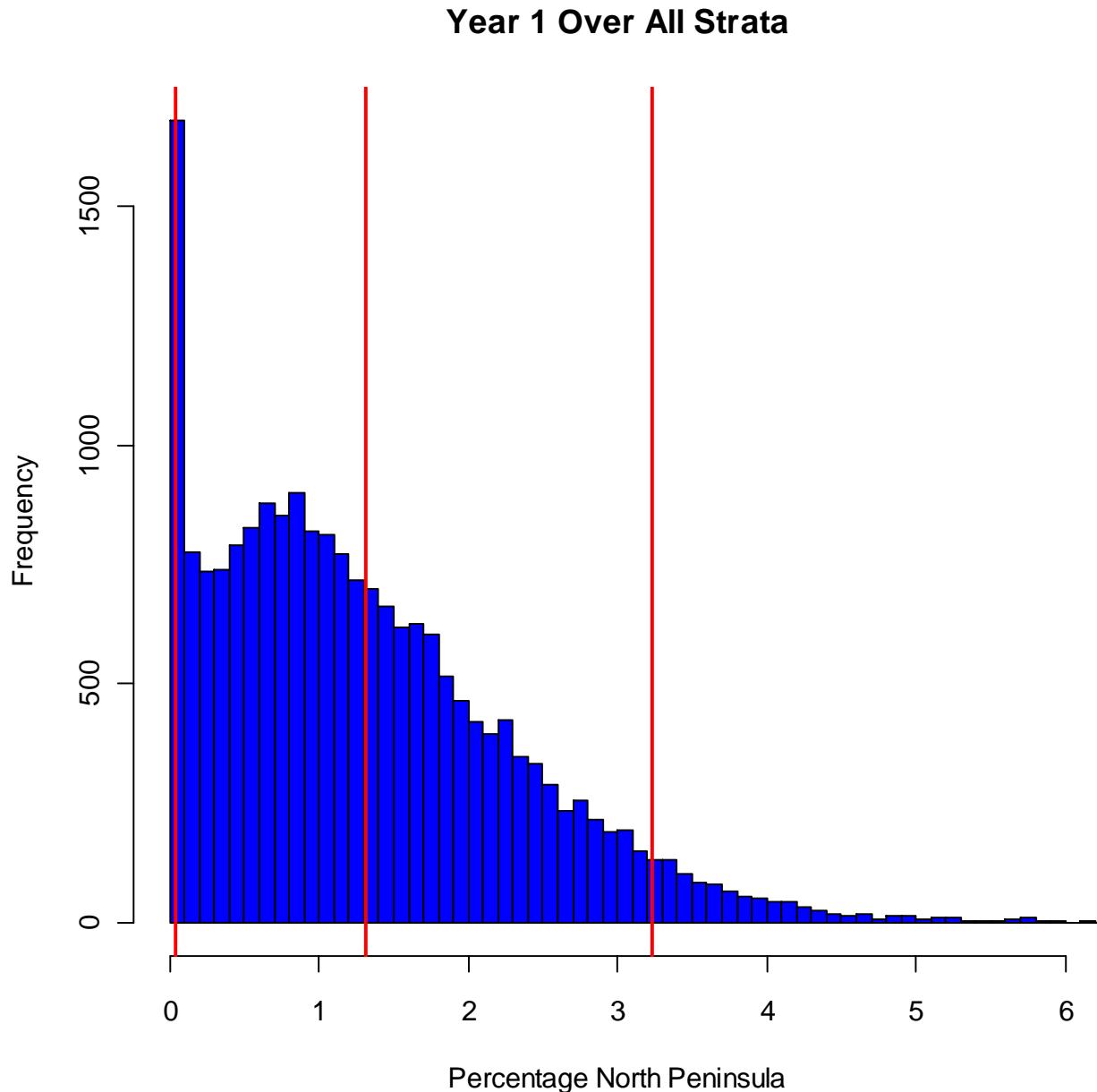
193 Table 3. Posterior means and Bayesian confidence intervals (90% CI) for the percentage of
 194 North Peninsula fish caught in the simulated harvest of sockeye salmon in the Ugashik District
 195 fishery over three years. Three levels of estimates were estimated: 1) individual estimates for
 196 each stratum in each year; 2) yearly estimates combining all strata in each year; and 3) overall
 197 grand estimate combining all years. As the level of the estimate increases, the confidence
 198 intervals get narrower.
 199

Level	Stratification	Mean	90% CI	
			5%	95%
Individual strata				
Year 1	Stratum 1	0.6	0.0	3.3
	Stratum 2	1.3	0.0	4.9
	Stratum 3	1.6	0.0	5.3
	Stratum 4	1.1	0.0	5.0
Year 2	Stratum 1	0.4	0.0	2.2
	Stratum 2	2.7	0.0	7.0
	Stratum 3	0.6	0.0	2.5
	Stratum 4	1.2	0.0	4.4
Year 3	Stratum 1	0.3	0.0	1.9
	Stratum 2	1.3	0.0	5.3
	Stratum 3	0.6	0.0	3.1
	Stratum 4	0.4	0.0	2.0
Yearly				
	Year 1-all strata	1.3	0.0	3.2
	Year 2-all strata	1.2	0.2	2.5
	Year 3-all strata	0.8	0.0	2.4
Across years				
	Over all years	1.1	0.4	2.0

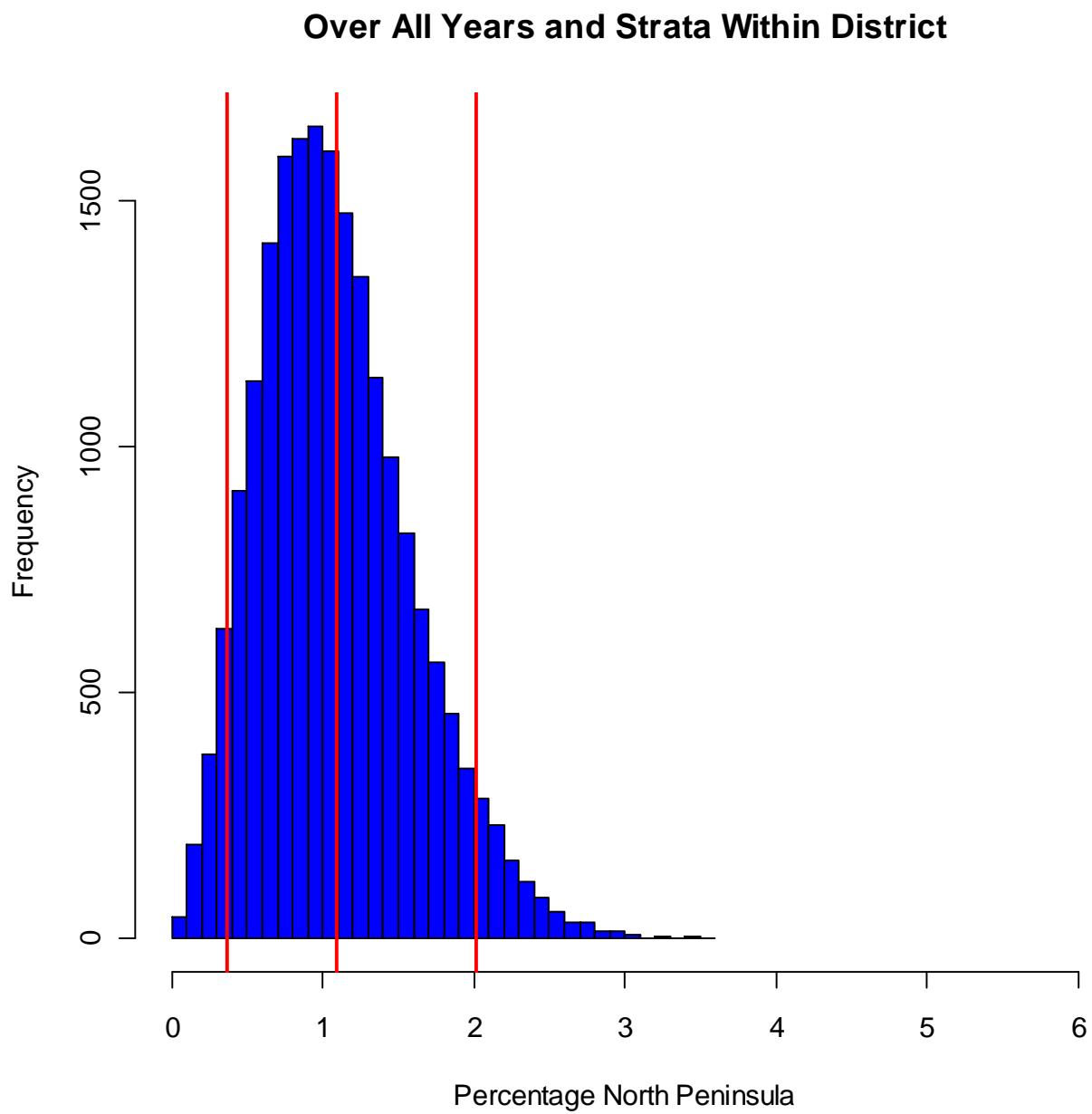
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202 Figure 1. Posterior distributions of North Peninsula's percent contribution to a simulated fishery
203 in the Ugashik District. Plots shown are for the four strata in Year 1 and are typical of those
204 observed in other years. Red vertical lines represent the mean and upper and lower bounds of a
205 90% confidence interval.
206



207
208 Figure 2. Posterior distribution of North Peninsula's annual percent contribution to a simulated
209 fishery in the Ugashik District. Plot shown is for Year 1 and is typical of those observed in other
210 years. Red vertical lines represent the mean and upper and lower bounds of a 90% confidence
211 interval.
212



213
214 Figure 3. Posterior distribution of North Peninsula's overall percent contribution to a simulated
215 fishery in the Ugashik District. Red vertical lines represent the mean and upper and lower
216 bounds of a 90% confidence interval.